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NETWORK APPLICATION IN INDUSTRY AND GOVERNMENT

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13. ABSTRACT

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A secondary purpose is to familiarize the reader with recent computational advances in the development of computer codes to solve these problems. For example, recent breakthroughs in the solution and human engineering aspects of minimum cost flow transshipment problems have made it possible to solve problems that require many hours of computing time with state-of-the-art commercial LP packages in only a few minutes. These increases in solution speed and reductions in computer memory requirements have made it practical to devise:

a) interactive on-line computer network solution systems linked with display and graphical terminal devices to enhance the applicability of mathematical programming.

b) network solution systems that are capable of solving vastly larger problems than previously imagined possible. For example, it is now possible to solve network problems with 50,000 nodes (equations) and 62 million variables.

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NETWORK APPLICATION IN INDUSTRY AND GOVERNMENT

by

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September 1975

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The primary purpose of this paper is to provide the practitioner with a short, but informative handbook of tools to use in modeling decision making situations as network flow problems. These tools are presented as part of the discussion of recent industrial and governmental applications. The intent is not to enumerate all applications of networks, but rather to give the reader a flavor of the versatility and usability of networks. An additional objective is to acquaint the reader with the process of visualizing a problem by means of network diagrams, thereby making it possible to capture important interrelationships in an easily understood "pictorial" framework.

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I. INTRODUCTION

Networks constitute one of the most significant classes of application problems in management science. Yet the extent of their pervasiveness has only recently come to be realized. A wide array of problems in production, distribution, financial planning, project selection, facilities location, resource management, and budget allocation - to mention only a few - fall naturally in the network domain [5, 6, 8, 9, 12, 15, 22, 27, 28, 29, 35, 36 37, 39, 40, 41, 43] .

The remarkable diversity and importance of network problems has become especially conspicuous in the past few years. This is primarily due to six main developments that have taken place since 1969:

(1) significant advances in solution methodology [3, 4, 16, 19, 21, 25, 26] ;

(2) improved computer software from intensive code development efforts [1, 2, 3, 4, 14, 17, 20, 23, 30, 38, 42] ;

(3) rigorous empirical studies to determine the best solution and implementation procedures [1, 2, 3, 4, 14, 17, 29, 20, 23, 30, 32, 38, 42] ;

(4) new modelling techniques [24, 35, 48] ;

(5) extensions of the network framework to broader classes of problems [24, 35, 38] ;

(6) application of these new tools and computer solution methods to substantive real world problems, leading to improved user-oriented software [1, 4, 20, 30] .

A direct consequence of these developments has been to enable network problems to be handled far more conveniently and effectively than in the past. In fact, the new computer codes have succeeded in solving network

problems with an efficiency many times beyond that of codes previously available. Problems that were "too large" or "too difficult" to accommodate even as recently as 1970 can now be handled on a routine basis. In fact, the fastest network code [1] is at least 15 times faster than any pre 1970 transshipment code and it requires much less computer memory.

The importance of these considerations may be further illustrated by the following example. Consider the situation in which the branch managers of an automobile manufacturing firm are required to develop a coordinated production and marketing schedule for the next quarter. Since these managers must deal with a multitude of details, the sequential and dynamic process of their deliberations proves to be extremely time-consuming and does not invariably lead to ideal results. Plans are met with counterplans in a tedious spiral of feedback and incremental change that only gradually converges to an agreeable resolution. Typically, time pressures force this process to end before a truly satisfactory operating policy can be decided upon. Because each proposed alternative customarily requires several days of calculation to measure its effect on company profit, the number of alternatives explored is often extremely limited.

With the appropriate use of an on-line network system (e.g., utilizing visual displays and appropriate simulation models) marketing and production personnel can discuss their goals and assumptions in a very short time. Answers to questions of the form "What if we do this?" can be quickly obtained and evaluated.

Because of the advances in solution codes both in terms of computer memory requirements and solution time, it is now possible to design and implement interactive on-line network optimization systems. In fact, we

have helped design two such systems, one for a major U.S. car manufacturer and another for a major U.S. bank, which illustrate the feasibility and practical utility of on-line applications.

Many organizations, however, are unaware of these advances. Significant opportunities to reduce costs, and/or increase returns have thereby been bypassed. Yet it is not difficult to identify key operating problems that can be profitably treated in the network framework. In fact, the use of such a framework often makes it easier to visualize important interrelationships that might otherwise go unrecognized, thus leading to the discovery of additional opportunities for improved operations. Accordingly, the purpose of this paper is to show how some major real world applications have been identified, visualized, and solved as network problems. (Mathematical justifications have been eliminated to keep the discussion informal and within manageable limits.)

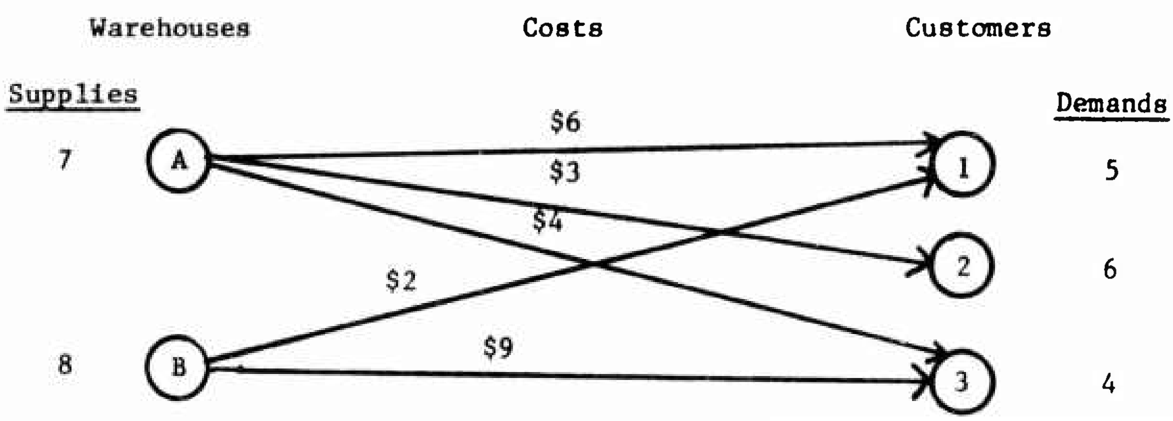
II. PURE NETWORK APPLICATIONS

Transportation Problem

A fundamental network structure that appears in numerous applications - either directly, or as a subproblem - is the "distribution" (or "transportation") problem, depicted in Figure 1.

Figure 1

Distribution (or Transportation) Problem



The "arrows" shown in this diagram are called arcs and the "circles" (which serve as endpoints of the arrow) are called nodes. In this example nodes A and B may be thought of a corresponding to warehouses and the nodes 1, 2, and 3 are corresponding to customers. The arcs indicate the possible ways to ship goods from the warehouses to the customers. Thus, for instance, the arc from node A to node 3 indicates that it is possible to ship from warehouse A to customer 3. The absence of an arc from node B to node 2 indicates that warehouse B cannot ship to customer 2.

In addition to the foregoing information, the distribution problem has three other elements which are generally common to network problems: supplies, demands, and costs. The supplies associated with the warehouse nodes in Figure 1 indicate that warehouse A has 7 units and warehouse B has 8 units of some commodity (e.g., drums of fuel oil, bushels of wheat, truck loads of refrigerators, ect.) to distribute to the customers. The demands associated with the customer nodes indicate that customer 1 requires 5 units of the commodity, customer 2 requires 6 units, and customer 3 requires 4 units. Finally, the cost attached to the arcs indicate that it costs \$6 per unit to ship from warehouse A to customer 1, \$3 per unit to ship from warehouse A to customer 2, etc. The objective in the distribution problem is to determine how much to ship from each warehouse to each customer in order to satisfy all supplies and demands and to minimize total cost.

In network problems, nodes typically represent entities, locations, periods or states, while arcs typically represent channels or routes for "shipping" from one node to another. A variety of the possible interpretation of nodes and arcs can in fact be illustrated by the simple network structure underlying Figure 1.

Treasury Application

A special "file merging" problem constitutes an important real world application with a transportation structure. The U.S. Department of the Treasury has two statistical data base files, Current Population Survey and Statistics of Income. These files are extensively used to analyze the effect of various policy changes (e.g., welfare payment levels, social security benefits, income tax rates, etc.) on federal revenue. However, to fully analyze the effect of these changes, it is necessary to have the information in both of these files. Thus it is desirable to have a method which limits the amount of information lost as these original files are merged.

The statistical files are weighted samples where control totals are maintained. In particular, each record contains a weight indicating the number of units (families) in the population which this observation represents. Thus, the file merging problem involves two interrelated problems. The first problem is to select the number and type of records to be in the merged file. The second problem is to select the weights for the merged records.

Figure 2 indicates how this problem can be formulated and solved as a transportation problem. In this diagram, the nodes represent records in each of the files to be merged. The supply or demand associated with a given node corresponds to the weight of the associated record. Since both files represent all U.S. inhabitants, the sum of the weights in each file must equal the total U.S. population. The "cost" coefficients are multi-attribute measures of the "distances" between the records in terms of information characteristics of the population subgroups making up these records. While there is not a unique procedure to use for calculating these

distance coefficients, the problem of determining distance between records in a microdata file is similar to the problem of determining distance between coordinates in a multi-dimensional space for multi-variate regression. Thus, a weighted sum of squared deviations can reasonably be used to estimate this distance (or information loss).

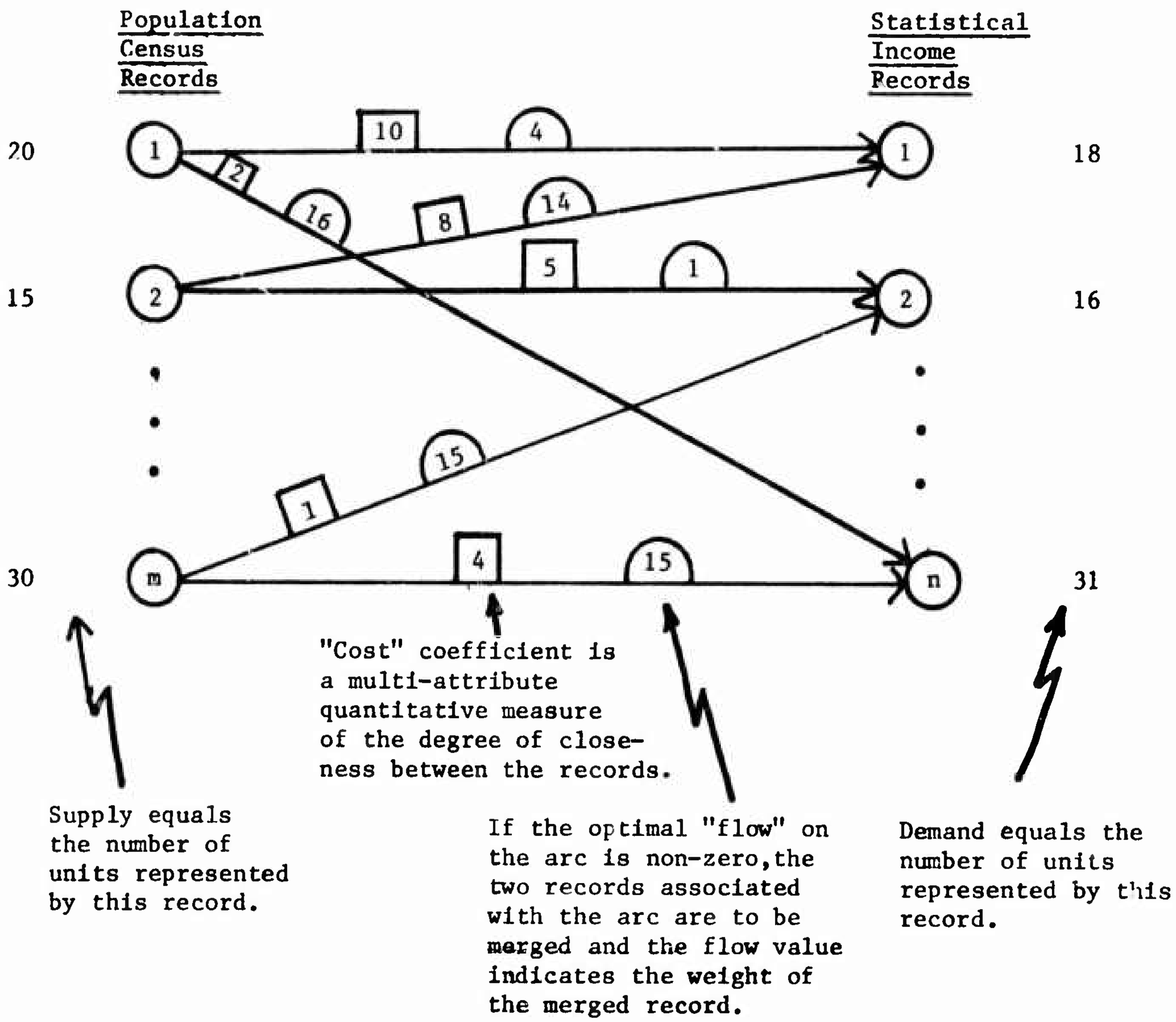
The solution to the transportation problem indicates: (a) the number of records to be contained in the merged file; (b) which records are to be merged with each other; and (c) the weight of each merged record. To illustrate, the numbers in the semi-circles in Figure 2 indicate the flows on the arcs. (The flow on an arc is the amount "shipped" from its initial node to its terminal node.) If the flow is non-zero, then the two records connected by this arc are to be merged. For example, record 1 of the population census file is to be merged with record 1 and record n of the statistical income file, creating two new records (one for each arc emanating from node 1 with non-zero flow). The weight (i.e., the number of units represented by each merged record) is equal to the flow on the arc. Consequently, solving the transportation problem yields the specifications for a merged file which minimizes the information lost with respect to the distance function used.

We have recently implemented an extended transportation system [1] for the U.S. Department of Treasury on a UNIVAC 1108 which is capable of solving a transportation problem with 50,000 nodes and 62.5 million arcs. This system will be used to merge the Current Population Survey file and Statistics of Income file. We recently solved a prototype of this problem with 5,000 nodes and 625,000 arcs in less than 4 minutes of central processing time and less than 9 minutes of total job time (including all input and output processing) on a UNIVAC 1108. We have also solved the problem on a

CDC 6600 in 10 minutes. (The total time on the CDC 6600 is slower due to the less efficient Control Data Corporation I/O library routines.) Thus, we are hopeful that the full size problem can be solved in a reasonable amount of time. The solution of the full size problem will mark the beginning of a new era of mathematical optimization and give a new meaning to the term large-scale optimization, bringing mathematical programming closer to its envisioned purpose of being a major vehicle for solving pressing real world problems.

Figure 2

Microdata Set File
Merge Problem



In problems of this magnitude, the intimate coordination of modeling and computer solution efforts are indispensible. In fact, the value of such coordination for problems at all levels can spell major differences in overall solution efficiency. For example, by coordinating the design of the solution system with the modeling effort in the above application, it was possible to devise an iterative procedure for bounding the optimal objective function value. This feature may be extremely important in solving the 62 million arc problem since a 95 percent optimal solution is sufficient for the Treasury's purpose and may require much less computer time.

The preceding transportation examples illustrate several of the basic features of network problems. However, further modeling considerations are required to accommodate features that are encountered in many real world applications. Additional types of flexibility in formulating network problems are given in the following examples. For instance, supplies and demands need not be imposed as "equality" restrictions, but can be expressed as upper and/or lower bound restrictions - e.g., supplying "at least" or "at most" a certain quantity. In general, a given supply or demand can be required to lie anywhere between two stated limits.

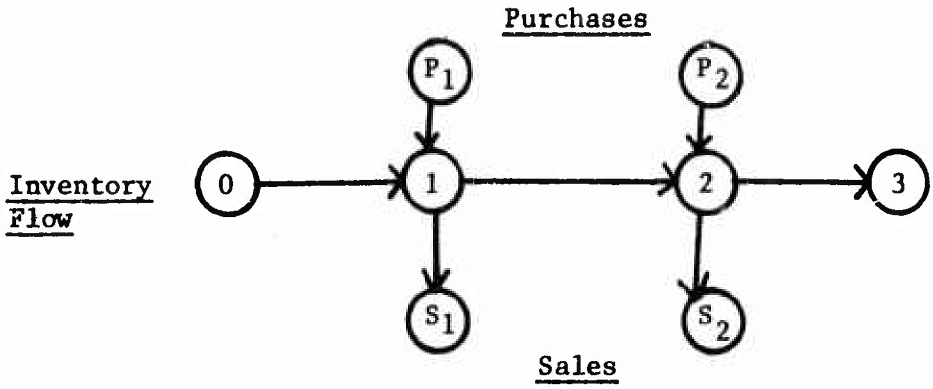
Another type of flexibility involves capacity restrictions on arcs. The number of units shipped across an arc - i.e., the arc flow - may be allowed to vary within specified limits (just as in the case of supplies and demands). For example, in the illustration of Figure 1, if warehouse A can ship at most 3 units to customer 1, then 3 constitutes a capacity restriction in the form of an upper bound (limiting the flow on the arc from node A to node 1). In the previous illustrations, a lower bound of 0 units of flow was assumed. Zero lower bounds are implicitly taken for granted in most applications, but sometimes nonzero lower bounds are imposed.

Transshipment Problems

Another class of models that typically exhibits an underlying network structure is the class of "inventory maintenance" models [43]. One instance of these is illustrated in Figure 3.

Figure 3

Inventory Maintenance



This model is time-phased. Node 0 represents the "starting point" at which some initial inventory level is introduced into the system (which may be treated as a supply at node 0). Nodes P1 and P2 represent inventory purchases in periods 1 and 2, and nodes S1 and S2 represent sales in periods 1 and 2. Nodes 1 and 2 which have arcs leading both to them and from them, are called <u>transshipment</u> nodes since the flow shipped to such nodes can in turn be shipped on to other nodes. (Problems with transshipment nodes are called <u>transshipment problems</u>.) For transshipment nodes that have no net supply or demand requirements of their own, which is the case here, the amount of flow received at the node must equal the amount shipped out of the node. Thus, the amount of flow on the arc from node 1 to node 2 equals the initial inventory, plus the amount received from period 1 purchases, less the amount that goes to period 2 sales - or in other words, this flow equals

the inventory remaining at the end of period 1 (or the beginning inventory for period 2). Similarly, the flow on the arc from node 2 to node 3 equals the ending inventory for period 2 (or the beginning inventory for period 3).

Data for the problem, if shown, would include expected sales volumes (demands at nodes S1 and S2), expected sales prices (revenues - or "negative costs" - on the arcs from 1 to S1 and from 1 to S2), expected purchase prices (costs on the arcs from P1 to 1 and from P2 to 2), and inventory holding costs (costs on the arcs from 1 to 2 and from 2 to 3). The objective is to determine how much to purchase and maintain in inventory in each of the time periods to maximize total profit (or minimize total cost). Additional considerations such as back order costs, quantity discounts on purchases and inventory capacity restrictions can also be incorporated into the problem.

It should be emphasized that in this example, and in many of the others cited, there may be a random or uncertain element in supplies and demands. Effective treatment of such uncertainties can be accomplished by one of three approaches [6, 9, 43] : sensitivity and postoptimality analyses, imbedded simulation, and "chance constrained" (or "stochastic") programming. Each of these approaches (which need not be mutually exclusive) reduces to solving or partially solving one or more ordinary transshipment problems, given that the original problem was modeled as a transshipment problem.

The inventory model of Figure 3 also applies to a number of problems which at first glance bear no particular resemblance to inventory maintenance. The scheduling of equipment purchases [9] and hiring of personnel [43] (as in the presence of strong seasonal demand swings), for example, can be amenable to formulation in a variant of this network model.

Car Industry Application

A transshipment problem which combines attributes of several of the preceding examples is the Production Planning and Distribution Model. This application, which is illustrated in Figure 4, is a bit more detailed than some of the preceding ones in order to disclose several useful features of the underlying model.

The problem represented by this diagram is that of determining the number of cars of each of three models (M1, M2, and M3) to produce at the Atlanta and Los Angeles plants (represented by the "Atl." and "L.A." nodes), and then to determine how many of each of these car models to ship from each plant to distribution centers in Pittsburgh and Chicago (represented by the "Pitt." and "Chi." nodes). The objective is to identify a production-distribution plan that minimizes total cost.

Bounded supplies are associated with the Atl. and L.A. nodes, indicating the least and most that can be produced at these plants. In addition, upper and lower bounds are placed on the various arcs emanating from these two nodes to control the minimum and maximum number of each particular car model that can be produced at these plants. Similar bounds (capacity restrictions) can be placed on other arcs. For instance, if there is a limit on the number of M1 type cars that can be shipped from Atlanta to Pittsburgh, then this would appear as an upper bound restriction on the "top-center" arc in the network. Finally, the number of each particular model required at Pittsburgh and Chicago is handled by placing bounds on the "far right" arcs. For example, if exactly 4,000 M3 - type cars are required in Chicago, then 4,000 becomes both the lower and upper bound on the M3 - Chi arc.

An interesting feature of this model is not only that it coordinates the production and distribution decisions, but also that it handles a multi-commodity problem in a "single-commodity" framework. That is, the three

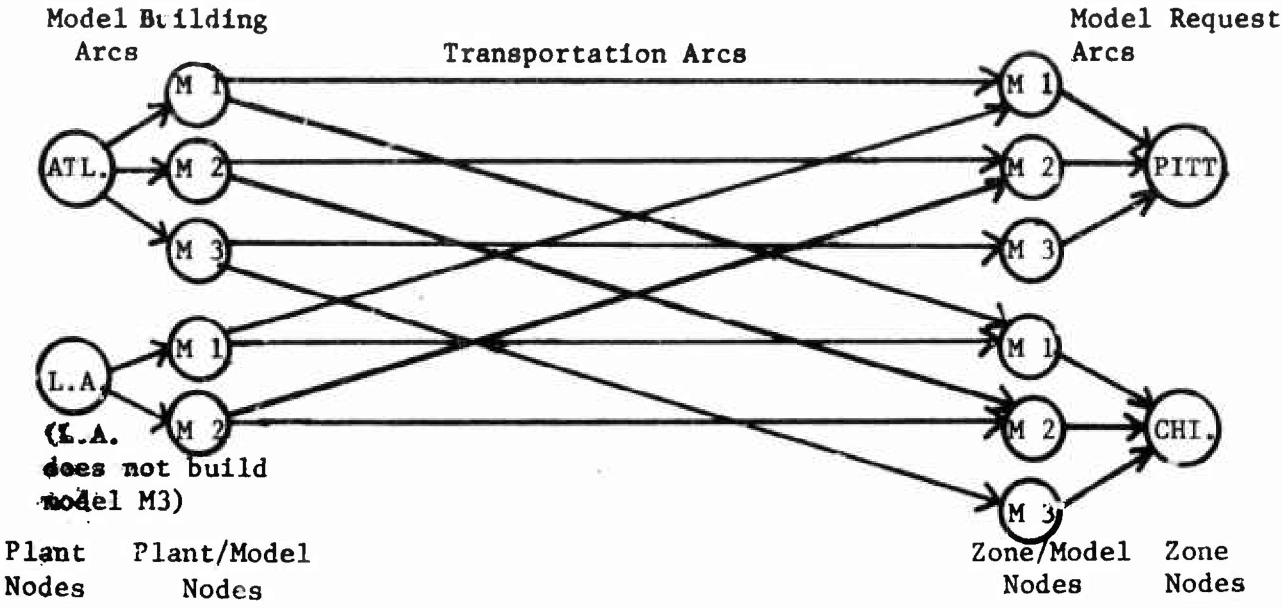
models M1, M2, and M3 are distinct commodities being shipped through the network, but their identities never get mixed or confused, as could be possible in some network models. This illustrates the importance of getting the "right" network formulation.

The typical size of this problem for a particular division (that is, Pontiac, Buick, etc.) is 1200 nodes and 4000 arcs. When we began working with the company on this problem, they were using a version of the SHARE out-of-kilter code [7] to solve these problems. The solution time ranged from 10 to 20 minutes on an IBM 370-145 and required 120 K bytes of computer memory. Using our transshipment code [1, 14] , such a problem can be solved in less than 10 seconds on a UNIVAC 1108, CDC 6600, IBM 360-145, or PDP-10. Only 23 K words or 92 K bytes of computer memory are required, thus making it easy to solve such problems in an on-line computer mode. In fact, due to the nature of the decision making environment of this application, the company has developed a on-line real time production planning and distribution system which is linked to a graphics display terminal and an English language input processor. This system is currently being used by the executive division for planning purposes.

Modified in various ways to handle distinguishing characteristics of different settings, this model can be used in production and distribution planning for many types of products other than cars. It may also be expanded to handle decisions relevant to various stages of a production process.

Figure 4

Production Planning and Distribution Model



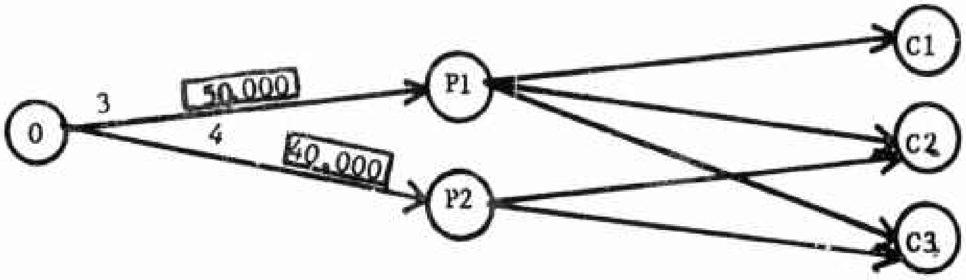
III. FIXED-CHARGE AND PLANT LOCATION PROBLEMS

Many problems involve certain "nonlinear" features in addition to the features illustrated in the previous examples. One of the most basic and prevalent forms of nonlinear problems is the fixed-charge network problem [3, 9, 43] whose major offshoots include the extremely important genre known as "location" problems.

A fixed-charge arc is one with the following special property: whenever the arc is "used" (i.e., permitted to transmit flow), a charge is incurred that is independent of the amount of flow across the arc. This property is illustrated in Figure 5.

Figure 5

Plant Location Problem



The flows on the arcs from node 0 to nodes P1 and P2 correspond to the number of units of a particular product that are produced at plants P1 and P2, respectively. In turn, the flows on the arcs from P1 and P2 to C1, C2, and C3 represent the amounts of the product that are shipped from P1 and P2 to each of the customers C1, C2, and C3. The plants P1 and P2, however, are not presently in existence, and must be built (or acquired) before the distribution to the customers can occur.

The figures in boxes on the arcs from node 0 to nodes P1 and P2 represent the fixed charges of building the two plants (50,000 for P1 and 40,000 for P2). Thus, if any positive flow occurs on the arc from 0 to P1 -- hence this number of units is produced at pl -- then the plant P1 must first be "built" and the full 50,000 must be paid. In addition, there is an "ordinary" cost for each unit produced at P1, and this is indicated by the 3 which appears on the arc from 0 to P1. Likewise, the numbers attached to the arc from 0 to P2 indicate that the fixed charge is 40,000 and the ordinary unit cost is 4 for this arc.

The fixed-charge distribution problem of Figure 5 lends itself to a variety of interpretations other than those involving "plants" and "customers". For example, such a network can be used to model decisions involved in purchasing or leasing equipment, hiring personnel, and so forth.

A fixed-charge network problem requires problem-solving "machinery" beyond that required to solve an ordinary network problem. However, the cost savings that can result from the solution of these problems generally far outweighs the increased machinery and computational effort required to solve them. Indeed, the advances cited earlier have made it possible to accommodate a wide class of difficult and previously unsolvable large-scale problems on a highly cost-effective basis [3] .

The fixed-charge framework also accommodates a variety of location problems. For example, P1 and P2 may be thought of as possible sites to locate warehouses or service centers which have fixed-charges attached to putting them in these locations. Thereupon the "residual" problem is an ordinary distribution problem of the type illustrated earlier in Figure 1. It is sometimes suggested that one way to deal with such problems is simply to itemize the number of possible ways to locate warehouses, and then solve the distribution problems remaining. However, with as few as 20 prospective sites, this would entail the solution of more than a million distribution problems.

Another location problem of the fixed-charge variety is the location of offshore oil drilling platforms. This problem incorporates the ability to select the placement of the drilling platforms, as well as determining which wells should be drilled from each platform once the platforms are placed. Figure 5 provides an instance of this problem by interpreting P1 and P2 to be prospective locations for platforms, and C1, C2, and C3 to be drilling sites.

The location of waste disposal collection centers in large cities, to which trucks from local collection areas bring refuse to be subsequently transported to main dumps, is likewise an instance of a fixed-charge location problem.

Cotton Gin Application

Agricultural problems often have network representations. A common problem, for example, is that of deciding how much of a particular crop to plant at various farms over a sequence of time periods, how much to ship for processing to various facilities (whose "activitions" involve fixed-charges) , and how much to store at various alternatives areas, leading up to final distribution. A real world problem which we recently solved provides a useful illustration [35] .

This application involves the minimization of total cost associated with processing or ginning cotton produced in the Rio Grande Valley of New Mexico and the Upper Rio Grande Valley of El Paso County, Texas [35] . This portion of the irrigated valley is approximately 95 miles in length and varies from .5 to 5.0 miles in width. The study area annually produces 40,000-55,000 bales of high quality Upland cotton and 10,000-18,000 bales of American Pima, an extra long staple variety. From historical cotton production data and aerial photos, the study area was divided into 150 production origins. Distance to the area's twenty different cotton gins scattered throughout the area were recorded. In recent years, the cotton production has decreased by 50 percent. Thus the objective is to determine an optimal policy of shipping cotton from farms to gins.

This project was funded under the provisions of the Hatch Act, whose purpose is to provide federal monies to improve efficiency of the domestic food and fiber production and marketing systems. The excess processing capacity of the ginning industry in the study area and the nature of the gin plant cost functions led industry people and researchers to question the efficiency of the industry. Many area cotton producers and industry personnel were willing to consider a reorganization of the industry if a

cost-cutting blueprint for change could be provided. A mathematical model was constructed to represent the entire system, since it was assumed that producers and gin firms would desire to implement a least-cost organization for the entire area industry rather than a fragmented industry solution. Implementation, to a considerable extent, is dependent on the anticipated cost savings of the least-cost organization. To convince farmers and gin plant operators that joint cooperative action is preferred to independent action, cost comparisons between present methods and alternatives suggested by the model were required. Implementation is being aided by recent Agricultural Stabilization and Conservation Service action which has funds available to make recourse loans on seed cotton in field storage. Personnel of the Cooperative Extension Service are involved in dissemination of study results and the educational process. It is visualized that the complete implementation of the solution will be an evolutionary process, extending over several seasons. However, without the model's solution any implementation of the fully cooperative procedures would be impossible. The evolution of a mathematical model for this evaluation is briefly described below.

The substantial decrease in cotton production in New Mexico in recent years has created a corresponding increase in excess plant capacity. Because existing processing capacity exceeds required capacity, it may be desirable to operate only a portion of the current gins. To evaluate this, the general form of a gin's cost function was sought. This disclosed that the cost function of a gin is typically a convex, piecewise linear function with a fixed charge. An example is given in Figure 6. The fixed charge in Figure 6 represents a one-time charge for activating a gin each year, since gins typically operate for at most 7 months a year, and include costs associated with electrical connection charges, cleaning, and salaried personnel.

The variable costs in Figure 6 are mainly due to maintenance and electricity, and do not include non-salaried personnel costs. The connection charge includes the purchase of an initial quantity of electricity. Consequently, the initial variable costs are lower than the regular variable costs in Figure 6.

Another ginning cost which is not represented in Figure 6 is regular time and overtime labor costs. In particular, the total weekly capacity of each gin is divided into the regular capacity and an additional capacity available with an overtime shift. Thus, there are two levels of weekly direct variable labor costs associated with each gin — one for regular time and another for overtime. If the capacity of the regular shift is exceeded, all of the additional cotton must be processed at this more expensive overtime rate. However, prudent use of this overtime may be profitable if it avoids the necessity of activating an additional gin.

In order to study the structure of this problem better, a small ficticious example of five farms which produce cotton available for shipping each week for three weeks will be used. The example assumes the existence of four gins which may operate for these three weeks plus an additional three weeks. Each gin has two levels of weekly costs for ginning. The first level is applicable to all cotton ginned during the regular shift, while the second level applies to all cotton ginned during the overtime shift. Also, each gin has a seasonal start-up cost and two levels of seasonal costs. The seasonal costs, the regular shifts' capacities with their associated costs, and the overtime shifts' capacities with the associated overtime costs are given in Table I. The production level for each of the five farms during the three weeks of cotton picking, the shipping costs from each farm to each gin, and the holding cost for storing cotton on the farms is also included in Table I.

Table I. Production, costs and capacities.

Shipping Costs

Farm \ Gin	1	2	3	4
1	6	2	4	7
2	3	5	8	4
3	2	6	2	9
4	5	3	7	6
5	4	5	9	4

Production (In Bales)

Week \ Farm	1	2	3	4	5	Total
1	15	40	30	15	15	115
2	35	75	45	50	50	255
3	20	60	20	35	20	155

Total Production 525

Weekly Gin Capacities (In Bales)

Gin	Regular Shift	Overtime Shift	Total	6-Weeks Total
1	20	10	30	180
2	15	10	25	150
3	20	10	30	180
4	50	20	70	420

Total Capacity 930

Gin Costs

Weekly Costs

Gin	1	2	3	4
Regular per-bale Cost	1	2	2	1
Overtime Per-bale Cost	3	7	5	2

Seasonal Costs

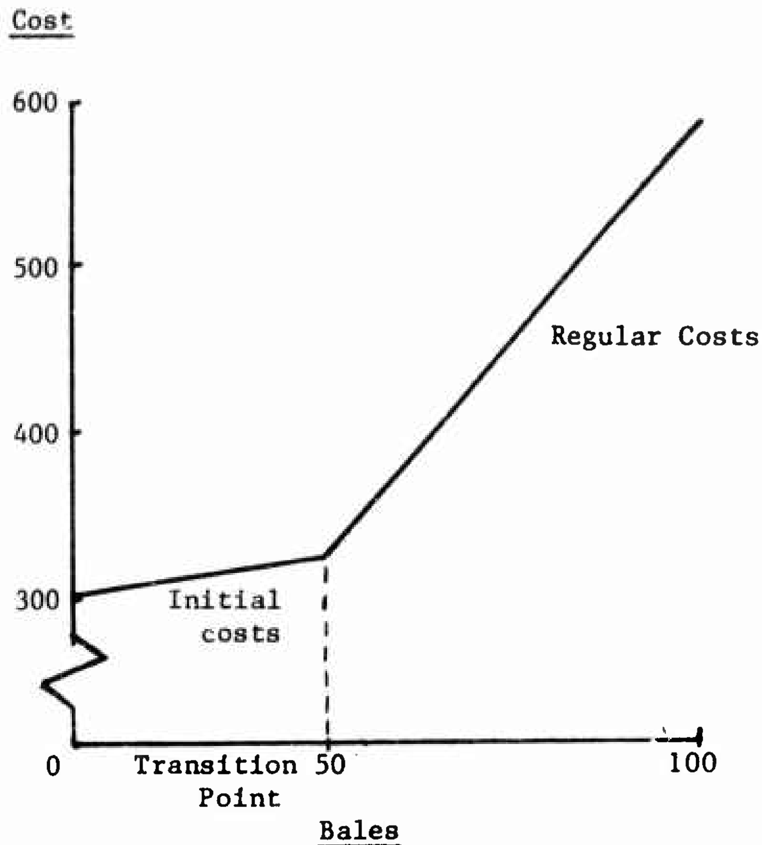
Start-up Cost	200	200	500	650
Initial per-bale Cost	1	6	2	1
Regular per-bale Cost	6	10	7	5
Transition Point between initial and regular rate in bales	40	20	30	70

Holding Cost

Cost per bale per week = 1

Figure 6

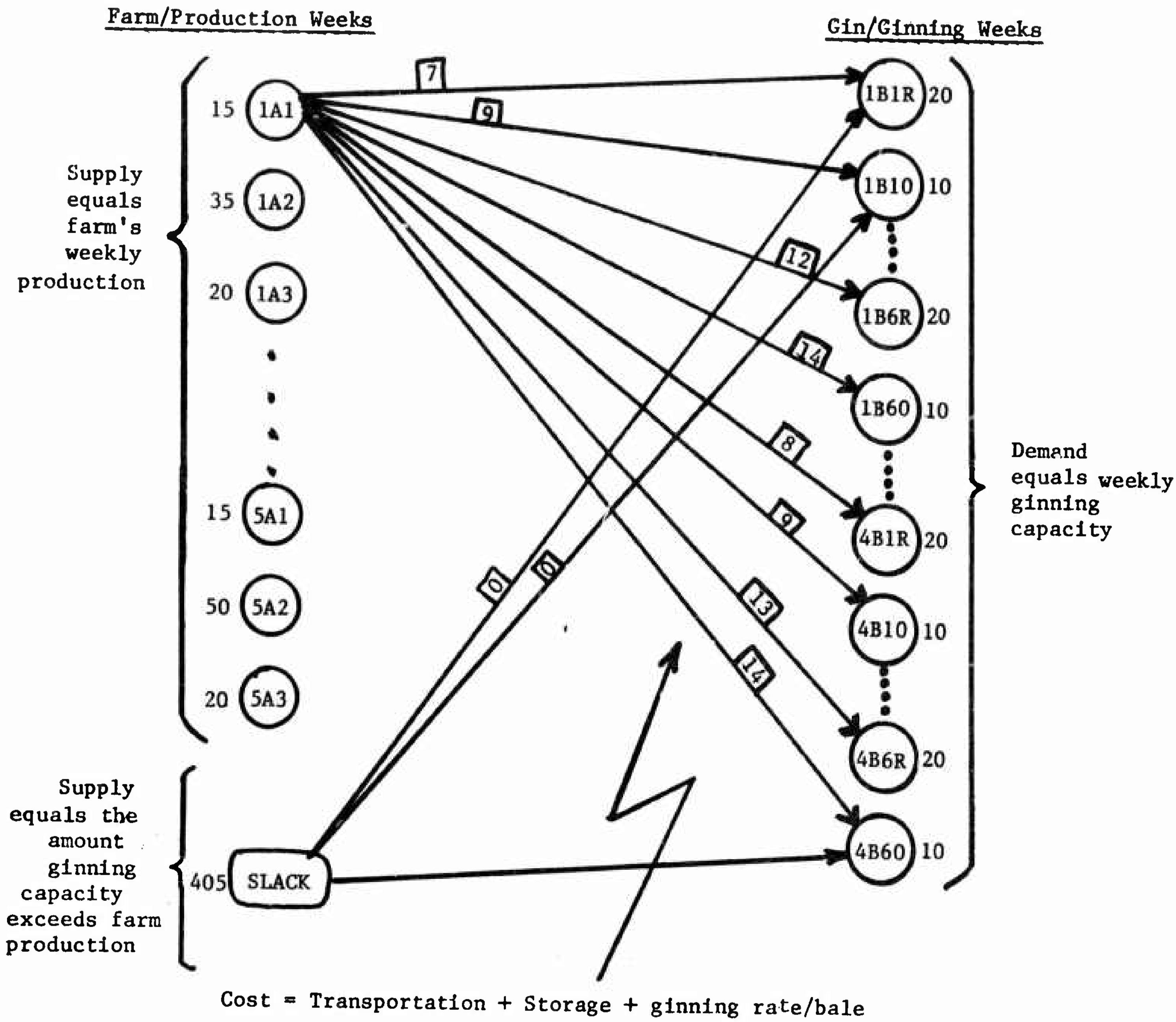
SEASONAL GINNING COSTS FOR GIN 1



This problem can be modeled as a transportation problem with extra linear constraints and zero-one variables. The resulting model, which is partially illustrated in Figure 7, is not solvable due to the existence of the extra constraints, the zero-one variables and the immense size of the problem. Fortunately, an equivalent fixed charge formulation was found for the problem which is solvable. Before describing the latter formulation, the original formulation will be briefly discussed to provide the reader with an idea of "do's and don'ts" in network modeling. Also this emphasizes

Figure 7

PARTIAL TRANSPORTATION FORMULATION



ANOTHER COST CALLED THE VARIABLE UTILITY RATE CANNOT BE ACCOMMODATED IN THIS TRANSPORTATION STRUCTURE.

again the desirability of finding not merely the "most visible" formulation of a problem, but the most effective one. However, the reader should note that any network formulation of a problem is likely to be useful. In fact, even capturing segments of a problem in a network structure can be useful because problems with embedded network structure can often be solved efficiently by repeatedly solving the embedded networks 6, 9, 23, 43 .

The transportation subproblem of the original model is shown in Figure 7. Farm i in week j is represented in Figure 7 by node iAj and gin i is represented for week j by two nodes iBjR and iBjO, one for regular ginning and another for overtime ginning. Thus the nodes 1A1 to 1A4 represent farm 1 for production weeks 1 through 4. Nodes 1B1R and 1B1O represent regular ginning and overtime ginning for gin 1 in week 1, respectively. Since the valley being studied involved 150 farms with 20 cotton production weeks and 20 gins with 30 ginning weeks, this transportation problem involves 4200 nodes plus an extra dummy supply node added to force total supply to equal total demand. Each farm node is connected to each gin node associated with a time period greater than or equal to the time period of the farm node. This yields approximately <u>2.46 million arcs</u>. The cost of each of these arcs is equal to the transportation per bale cost plus storage/bale and ginning/bale cost. This transportation formulation does not accommodate the fixed charge or convex cost aspects associated with the seasonal (total production) ginning cost function of Figure 6. Consequently, as previously noted, the full formulation of the problem involves extra linear constraints and zero-one variables which yields a mixed integer linear programming problem whose scope exceeds current solution capabilities.

The improved formulation is based on the idea of treating each of the problem's cost components separately. There are nine main elements which affect the optimal solution to this problem:

(1) The cost of shipping cotton from each firm to each gin.

(2) The start-up cost for each gin.

(3) The capacity of each gin's regular shift and its overtime shift.

(4) The variable weekly costs of ginning at regular rates.

(5) The variable costs of ginning at overtime rates.

(6) Each farm's holding costs for storing cotton.

(7) The variable initial utility rates. (See Figure 6.)

(8) The variable regular utility rates. (See Figure 6.)

(9) The transition point in seasonal gin production between initial and regular rates. (See Figure 6.)

By separating the cost elements, the problem may be modeled as the fixed charge transshipment problem depicted in Figure 8. The area enclosed in double lines in Figure 8, which considers only the arcs from farm one to gin one, is shown more clearly in Figure 9.

First consider the nodes of the transshipment model. There is an origin node representing each farm for each of the ginning weeks. In Figures 8 and 9 the iAj nodes represent farm i in week j. The supply of each of these nodes is the estimated amount of cotton to be picked at the farm during this week. The iBj nodes of Figures 8 and 9 represent gin i in week j. The total amount of cotton processed by each gin during the year is then channeled through a single node, called the weekly master node for the gin. Node iC represents the weekly master node for gin i in Figures 8 and 9. This transshipment structure eliminates the need for creating two arcs from each farm

node to accommodate the two weekly ginning costs, but adds the four weekly master nodes. Another four nodes are also added and linked to the iC nodes to accommodate the variable seasonal utility costs. These nodes are represented by the D level nodes in Figures 8 and 9. Finally, all flow is channeled through a single node, node E, which acts as a sink for the entire production. By setting its demand equal to total farm production, the need for the slack node and arcs in Figure 7 is eliminated.

Next consider the farm arcs of the transshipment model. For each week the cotton picked on a farm for that week may be shipped to any one of the four gins or may be stored at the farm. Shipping to the gins is represented by four arcs (one to each gin), while storage at the farm is accomplished by shipping the cotton to the source node of that farm for the subsequent ginning week. The effect of storing the cotton is to increase the amount available for shipping next week. The reader should compare Figures 7 and 8 and note the dramatic reduction on the number of arcs by handling cotton storage in their transshipment manner. For example, in a problem with 150 farms, 20 production weeks, 20 gins, and 30 ginning weeks, the transportation formulation uses over one million more arcs to model the storage feature than the transshipment formulation.

The gin arcs may be used to handle both the weekly capacity restriction and the overtime capability of a gin. Specifically, two arcs are used to link each iBj node to its iC node. One arc has a cost equal to the labor cost of the regular time shift and an upper bound equal to the weekly regular time production capacity of the gin. The other arc has the appropriate costs and capacity of the weekly overtime shift. Note that since the two arcs ship between the same two nodes, the arc with the higher cost will not be used until the arc with the lower cost has reached its capacity; this

insures that no overtime is used until the capacity of the regular shift has been reached. Additionally, observe the marked reduction in the number of nodes and arcs used by the transshipment formulation to model the overtime aspect.

From each gin's weekly master node, C, there are two more arcs leading to the gin's seasonal master node, D. These two arcs represent the initial and regular seasonal ginning costs depicted in Figure 6. The arc associated with the initial costs has an upper bound which is equal to the transition point between initial and regular utility costs of the gin. This feature of the transshipment model eliminates the extra constraints associated with the transportation formulation.

Finally, the single arc from each gin's seasonal master node to the sink node are the fixed charge arcs . That is, if any flow occurs on these arcs (i.e., if the gin is used at all) the full fixed start-up cost of Figure 6 is paid.

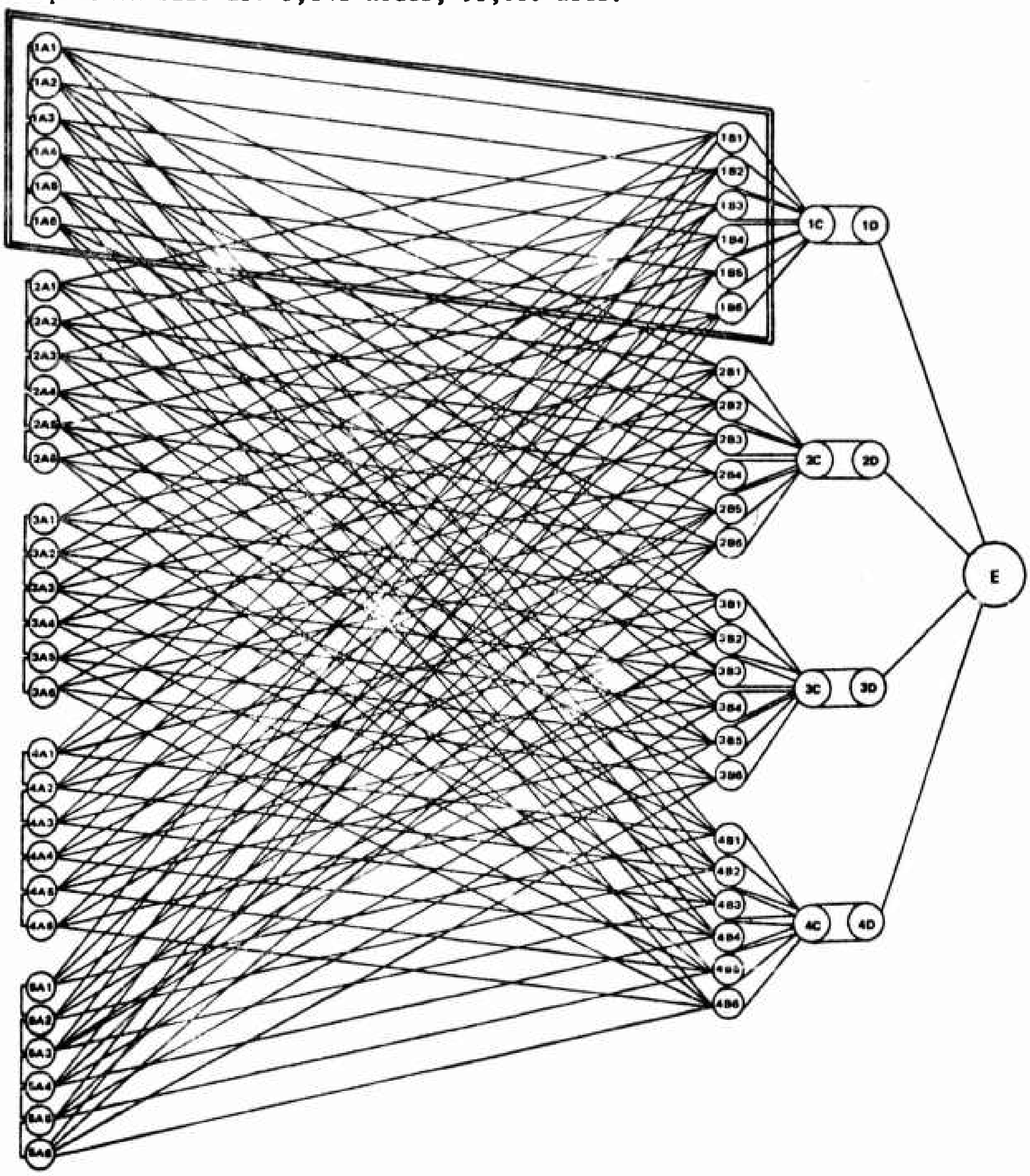
This formulation for the 150 farm problem reduces the number of arcs from 2,460,600 to 95,610. It does, however, increase the number of nodes from 4,201 to 5,141. Such a fixed charge transshipment problem is well within the solution capability of our state-of-the-art fixed charge network code [35] . However, by exploiting additional structural features of the problem, this transshipment model can be compactified to 3,441 nodes and 61,640 variables. For brevity, these refinements are not included.

To solve this problem a branch and bound program was designed [35] where the branching variables correspond to the fixed charge (node D-E) arcs. Then each node in the decision tree corresponds to a transshipment problem. This code solved a cotton gin problem with 991 nodes, 16,981 arcs, and 15 fixed charge arcs in 5 minutes on a CDC 6600 and a problem with 3,441 nodes, 61,640 and 20 fixed charge arcs in 50 minutes on a CDC 6600. The solutions

Figure 8

NETWORK DIAGRAM FOR THE TRANSSHIPMENT FORMULATION

For 150 farms, 20 production weeks, 20 gins, and 30 ginning weeks, the problem size is: 5,141 nodes, 95,610 arcs.



indicated that the farmers could obtain substantial savings (more than a 20 percent reduction in ginning costs) by closing some gins and working as a cooperative.

IV. GENERALIZED NETWORKS

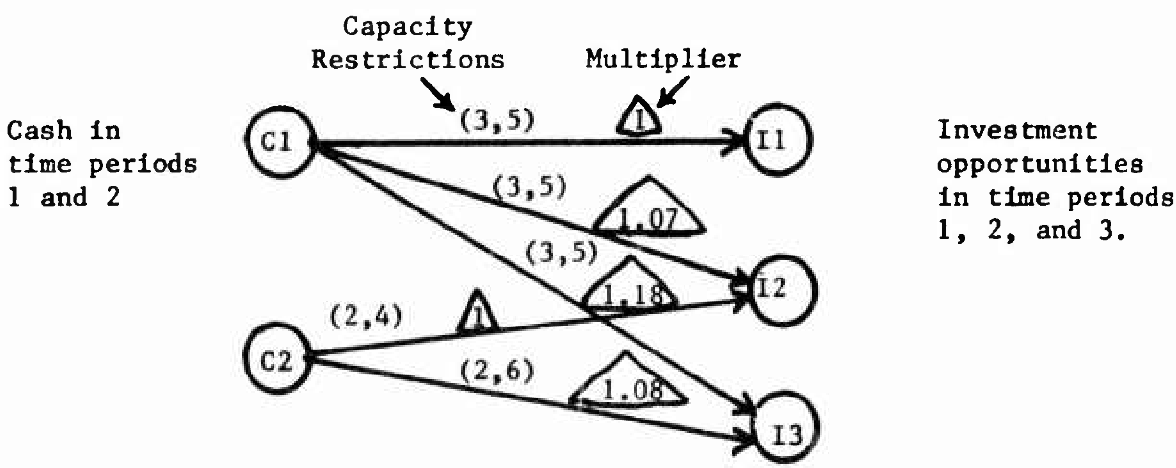
In the transportation and transshipment problems that have been discussed thus far, flow is not changed as it passes across an arc. That is, one unit of flow starting across an arc is still one unit of flow after crossing the arc. Such problems are often called pure transportation and pure transshipment problems or more simply pure network problems. A different form of network problem involves "flows with gains and losses". In this type of problem, the amount of flow that enters an arc may differ by a specified multiple from the amount that leaves the arc. Transmission through electrical power lines is a good example of this phenomenon, where the power deteriorates over distance. The cash flows of financial transactions constitute another prominent example. For example, interest charges on borrowed money decrease the amount of cash as it travels across an arc from a present time period to a future time period. On the other hand, interest income increases the cash value of a bond, whose possible redemption at alternative future periods may be represented by arcs leading from the purchase period to each of the alternative redemption dates. In a like manner, devaluation (or appreciation) of certain types of inventories over time can be represented by arcs whose flows attenuate (or amplify) by specified multiples. Additionally, the design of sewage treatment plants can be viewed in this fashion since the effluent passes through a sequence of purification processors with varying efficiencies. Arcs which have this character are called generalized arcs, and networks which include them are called generalized networks.

An illustration of a problem involving generalized arcs is the cash management model depicted in Figure 10. In this figure, node C1 represents cash in period 1 and node C2 represents cash in period 2. Investment possibilities in periods 1, 2, and 3 are denoted by the nodes I1, I2, and I3, respectively. There is no arc connecting node C2 to node I1 because a period 1 investment is not available in period 2 within the context of this example. However, it is possible to commit current cash to a future investment, and hence arcs are included that lead to future investment possibilities. Such a commitment may be carried out, for example, by temporarily putting the money in the bank or in standard "interim" investments which yield interest earnings until the targeted investment can be made.

The numbers in the "triangles" on the arcs indicate the amount that each unit of flow becomes as it traverses the arc. Thus, the 1.18 in the triangle on the arc from node C1 to node I3 indicates that this arc is a generalized arc whose "multiplier" is 1.18. Consequently, whatever the flow across the arc, 1.18 times this flow is actually transmitted to node I3. For example, if the flow on the arc is 4, then node I3 receives 4(1.18) = 4.72 units of flow. Finally the numbers in parentheses represent lower and

Figure 10

CASH MANAGEMENT



upper bounds on how much cash from a given period is permitted to be invested in each alternative. For brevity the supplies, demands, and costs are not shown. Note that this illustration explicitly introduces the time element into the network context. The time-phased purchase of equipment and inventories can be put in a similar framework.

The cash management model of Figure 10 can readily be enlarged to include sources of funds in addition to cash, such as maturing accounts and notes receivable, sales of securities, borrowing, etc., and uses of funds other than a single "investment", such as maturing accounts and notes payable, purchases of securities, lending, etc. Indeed it is possible to incorporate discount, interest charges, and other related financial considerations directly into the model [22] . The objective, as in earlier examples, is to minimize the cost or maximize the profit of the transactions involved.

There are numerous applications which can be formulated and solved as a generalized network. These include structural design problems [6] , machine loading problems [6, 9, 43] , blending problems [6, 43] , the caterer problem [9, 43] , and scheduling problems [6, 9, 43] such as production and distribution problems, crew scheduling, aircraft scheduling, and manpower training.

Mathematically, it is considerably harder to devise efficient specialized algorithms and computer codes for solving generalized networks than for pure networks. The most efficient generalized network computer codes require 3 to 4 times the amount of time to solve generalized networks as pure network codes require to solve pure networks of the same size. Nevertheless, computer codes [19, 20, 21] which can solve generalized capacitated and uncapacitated transportation and transshipment problems with thousands of nodes and variables have recently been developed. Current solution

times on pure transshipment problems of the same size 1 are typically 10 seconds on a CDC 6600. It is important to note, however, that the generalized network times are at least 100 times faster than state-of-the-art LP codes on these problems 20 .

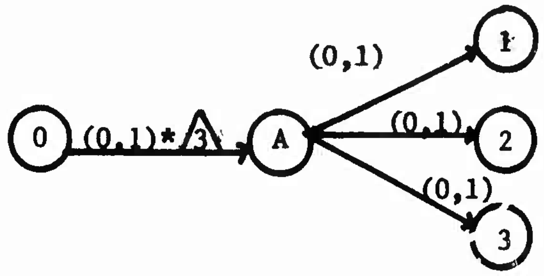
V. INTEGER GENERALIZED NETWORKS

The uses of generalized arcs just discussed do not by any means exhaust their range of application. In fact, upon adding the requirement of discreteness, which compels the flows on particular arcs to occur in integer quantities, the generalized network problem is capable of modeling an unexpected diversity of problems. For example, introducing multipliers into the type of model illustrated in Figure 1 produces a framework for problems such as scheduling variable length television commercials into time slots, assigning jobs to computers in computer networks, scheduling payments on accounts where contractual agreements specify "lump sum" payments, and designing communication networks with capacity constraints. While these are "direct" applications, there is a modeling principle which enables even somewhat more complex applications to be handled in an entirely straightforward way. Figure 11 provides an illustration of this principle. The 0 and 1 in parentheses associated with each arc in Figure 11 indicate the lower and upper bound on the flow across the arc. In addition, the 3 in the triangle on the arc from node 0 to node A indicates that this arc is a generalized arc whose "multiplier" is 3. Thus, whatever the flow across the arc, 3 times this flow is actually transmitted to node A.

Moreover, the asterisk on the arc from node 0 to node A indicates that its flow must be an integer amount. Since the bounds on the arc constrain

Figure 11

GENERALIZED NETWORK WITH INTEGER FLOW RESTRICTIONS



the flow to lie between 0 and 1, and the integer requirement rules out all "fractional" values, the only acceptable flow values are exactly 0 and 1. If the flow is 0, then $3 \cdot 0 = 0$ and no flow gets transmitted to node A. But if the flow is 1, then 3 units are transmitted to node A. Further, because of the upper bounds of 1 on each of the three arcs leaving node A, the only possible way to distribute the 3 units flowing into node A is to send exactly one unit to each of the nodes 1, 2, and 3. Thus, by the devise of giving all arcs bounds of 0 and 1, and introducing a generalized arc, the following effect has been achieved: when the flow on the arc from node 0 to node A is 0, the flow on each of the three arcs out of node A is 0; when the flow on the arc from node 0 to A is 1, the flow on each of the three arcs out of node A is 1. This effect makes it possible to model a variety of problems. One example, is the Undergraduate Flight Training (UFT) model on which we are currently conducting a feasibility study for the U.S. Air Force.

UFT Graduate Application

Undergraduate Flight Training (UFT) graduates are required upon graduation to take advanced flight training and survival training courses enroute to their first operational assignment. The purpose of the advanced flight

training is to qualify a pilot for a specific aircraft. Advanced flight training is offered only in formal schools usually by the Major Air Command, the principal aircraft user. Newly qualified UFT graduates will additionally require from one to four extra training courses before being assigned to a crew - e.g., basic survival (Washington), water survival (Florida) , air weapons delivery (Texas), etc. These courses are only offered at certain times, have enrollment limits, and may have prerequisites. The identification of schedules is further complicated by attendance requirements at Combat Crew Training courses, various modes of transportation, the number of dead days in the pipeline, and the opportunity for the UFT graduates to take leave as desired, etc.

To solve this UFT graduate scheduling problem, the Air Force developed a computer program (called the UFT Pipeline Scheduling Model) which generates from one to five feasible least cost schedules for each graduate. Using these schedules and course enrollment limits, the personnel manager in the Training Pipeline Management Division manually assigns each graduate to one of his feasible schedules. Clearly, this is a difficult and time-consuming task to do by hand; further, the total cost of these manual assignments may be far from optimal. Thus, the Air Force is interested both in automating this assignment procedure and in obtaining an assignment schedule which minimizes total cost.

The UFT problem may be modeled using arcs such as the generalized arcs of Figure 11. Here, a generalized arc with a multiplier equal to the number of classes in the schedule is created for each schedule of classes that a particular student pilot might take. In Figure 12, these are the arcs from the man nodes to the schedule option nodes. The arcs emanating

from a schedule option node in Figure 12 lead to the individual classes making up the schedule. Since these arcs have an upper bound of one, if a particular schedule is "selected", then every class in the schedule is also automatically selected, which,of course,is precisely the relationship desired. The objective of the model is to pick a schedule for each student that will minimize the value of the overall class assignment, subject to satisfying the upper and lower attendance limits for each class expressed as bounds on the class node-sink arcs of Figure 12. The costs on the arcs are thus all equal to zero except on the man-schedule arcs. These arcs have a cost equal to the cost of this schedule.

We developed a branch and bound network code to solve the UFT model that succeeds in solving problems involving 120 men, 460 schedules, and 200 classes in 10 seconds on a CDC 6600.

Equivalent Problem Types

The generalized "set covering" and "set partitioning" problems can also be accommodated by the modeling technique illustrated in Figures 11 and 12. These problems include airline crew scheduling problems, emergency fire station location problems, and many others. An important member of this class of problems is the Acquisition problem depicted in Figure 13.

In this diagram, nodes A, B, C represent different types of acquisition possibilities (e.g., different types of specialized equipment, trained personnel, pollution control devices, real estate investments, etc.) and nodes 1, 2, 3, 4 represent functions or qualities that must be exhibited by the total set of items acquired. Thus, for example, in the equipment acquisition context, nodes 1 through 4 may represent various kinds of processing capabilities. Alternatively in the real estate investment context,

Figure 12

UFT MODEL

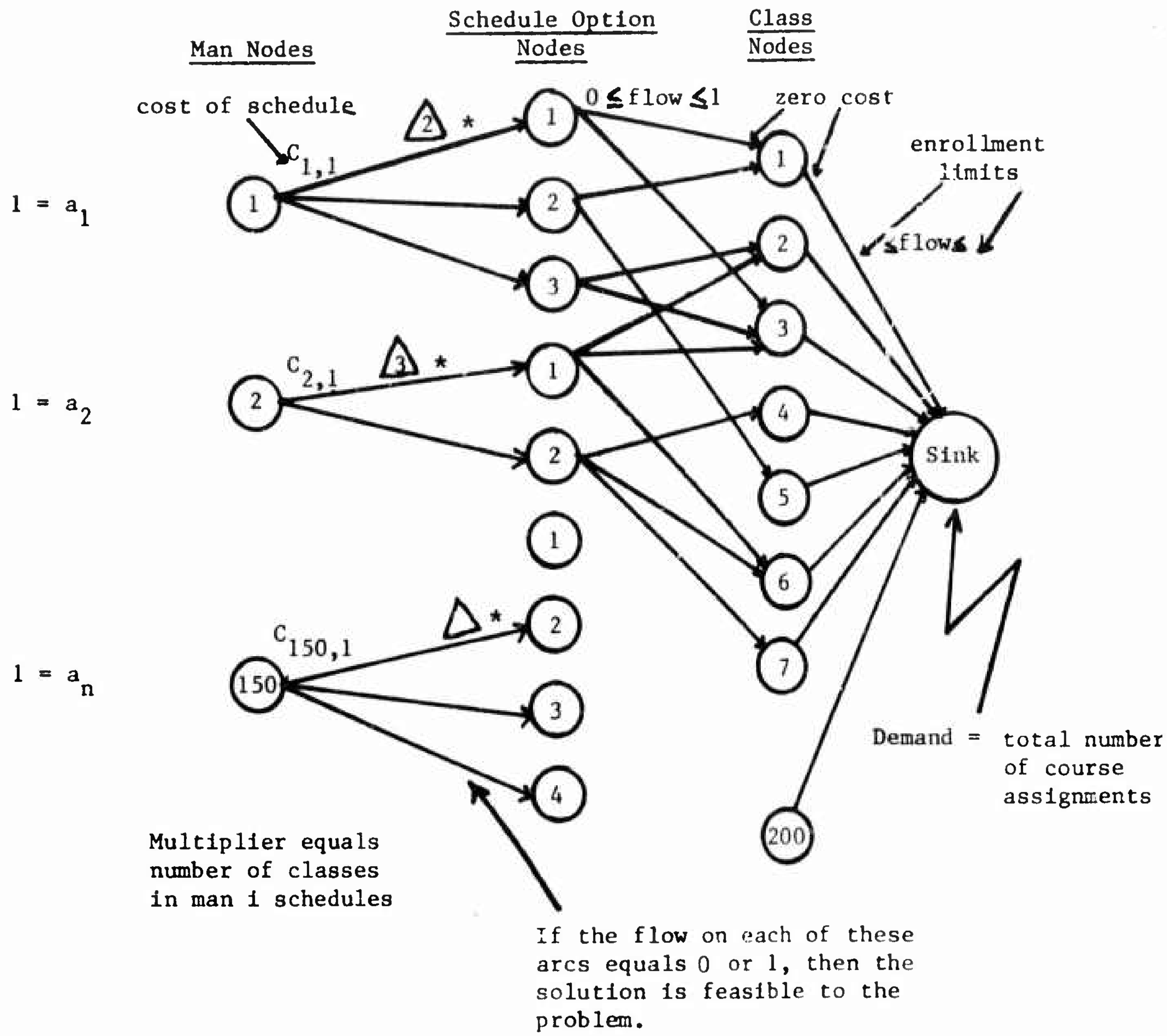
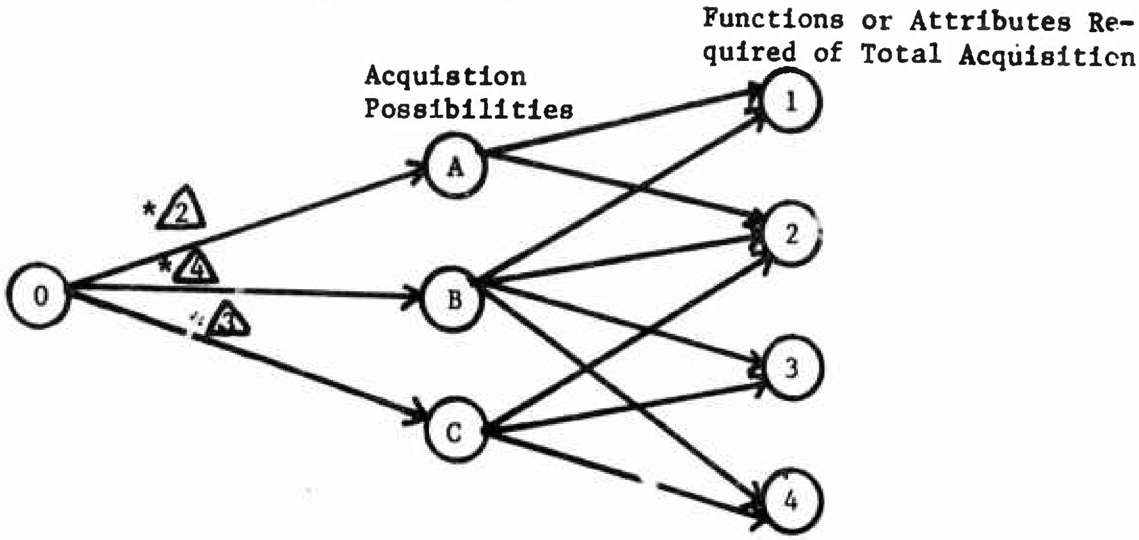


Figure 13

ACQUISTION PROBLEM



these nodes might represent attributes such as geographic locations, types of land, etc.

The arcs from nodes A, B, C to nodes 1, 2, 3, 4 identify the exact functions that each of the potential acquisitions A, B, and C are able to accommodate. In particular A accommodates functions 1 and 2, B accommodates all functions, and C accommodates functions 2, 3, 4. The "multipliers" appearing in triangles on the arcs from node 0 to nodes A, B, and C, respectively equal the number of different functions that each of A, B, and C, individually are capable of handling. Asterisks attached to these arcs indicate the integer restriction as in Figure 11, and all arcs in the diagram are assumed to have lower and upper bounds of 0 and 1, respectively. Given demands of "at least 1" on each of the function nodes 1 through 4, and costs on the generalized arcs equal to the acquisition cost of A, B, C, respectively, the model depicts the problem of determining the least cost set of acquisitions whose members can collectively handle all required functions or exhibit all required attributes.

Other modeling techniques involving integer-restricted generalized arcs make it possible to accommodate problems in which flows on selected arcs must be in multiples of some specified constant, as in financial contexts where investments and sales (e.g., of bonds) are permissable only in restricted denominations.

Still more generally, by allowing multipliers on all arcs, it is possible to model any zero-one linear programming problem as an integer generalized network problem |24| . The above examples are special instances of the general technique used to accomplish this transformation.

VI. GENERALIZED ASSIGNMENT MODEL

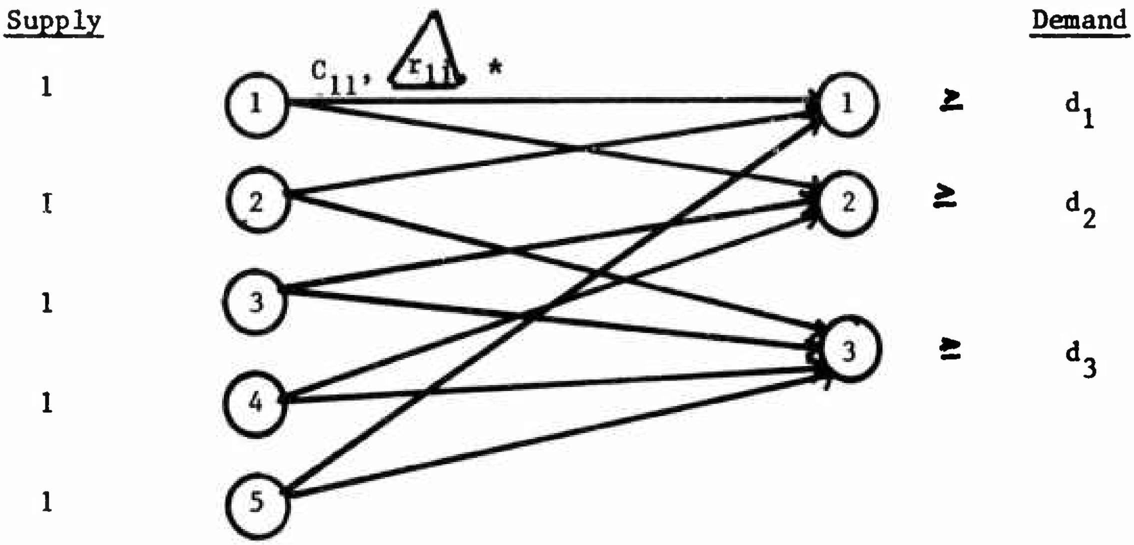
A cousin to the integer generalized network problem is the so-called generalized assignment model. In fact, a generalized assignment model is simply a special case of the generalized integer network problem. In these problems there are only two sets of nodes. One set (origin nodes) only has arcs leading out of the nodes and the other set (destination nodes) only has arcs leading into the nodes. See Figure 14. Each origin node has a supply of exactly one and each destination node i has a demand of at least (or at most) d_i. Additionally, each arc has a "cost" and a multiplier and the flows on each arc must be integers. For example, c_{11}, r_{11}, and on the arc between origin node 1 and destination node 1 of Figure 14 indicate the cost, multiplier, and integer requirement on the arc.

The name "generalized assignment model" is due to its interpretation as assigning personnel (origin nodes) to jobs (destination nodes) where each person contributes a specified amount (multiplier value) to a job. Further,

each person can only be assigned to one job (integer requirement). In contrast to the classical assignment problem [6, 9, 43] , however, a job may have several persons assigned to it in order to satisfy its demand.

Figure 14

GENERALIZED ASSIGNMENT PROBLEM



Another application visualizes the origin nodes as checking accounts and the destination nodes as days of the month. In this application the demands are "at most" requirements ($\leq d_i$) and represent daily auditing capacity. Each account must be assigned to a day of the month and the multipliers represent the auditing effort required for the account.

A further application of this model involves the assignment of ships to shipyards for overhaul. In this application, the origin nodes in Figure 14 represent the ships and the destination nodes shipyards. Like the last application, the deamnds are "at most" requirements and represent shipyard capacity in days. The multiplier represents the number of days required for the overhaul at that shipyard. The "cost coefficients" could be a weighted

combination of attributes such as transportation costs, overhaul costs, naval desirability of assigning the ship to this yard, etc. The objective would then be to minimize total "cost".

This problem is a subproblem of a model designed by Gross and Pinkus [27] to evaluate shipyard expansion questions. Their objective was to minimize total overhaul time; that is, minimize $\Sigma\, r_{ij}x_{ij}$, where x_{ij} is the flow on arc (i,j). Gross and Pinkus noted that these subproblems have a "nice" mathematical structure and, thus, an efficient solution approach could probably be devised. This stimulated Ross and Soland [39] to design a special code to solve generalized assignment problems using a Lagrangean relaxation approach. The code of [39] has proved to be extraordinarily fast, solving problems with 1000 ships, 20 yards, and 20,000 arcs in less than 30 seconds on a CDC 6600.

VII. CONSTRAINED NETWORK PROBLEMS

The last type of network problem to be discussed in this paper is a problem which contains a network structure, but which also has some additional linear constraints. The first formulation of the cotton gin problem is an example of such a problem. As illustrated in the cotton gin application, it is often possible to incorporate such constraints directly into the network structure [6, 9, 13, 34, 43] . However, there are a number of problems where this is not possible. The following manpower assignment formulation provides an example of such a problem.

Manpower assignment problems typically involve the assignment of personnel to tasks in the "most effective manner". Normally there are several competing factors involved in making an assignment [5] . Consequently, the

decision maker is faced with determining the best compromise of the competing goals subject to satisfying the tasks. There are several ways to formulate such a multi-attribute, multi-goal assignment problem [5] . A formulation which has proven to be quite good is given below.

The manpower assignment problem depicted in Figure 15 has the objective of determining the cheapest way to assign person A and person B to any two of the three jobs 1, 2, and 3. Here the supply of 1 unit at the personnel nodes indicates that each person must be assigned to a job. The demands are at most 1 unit, so that each job will receive at most one person. Important additional goals are that the utility of the job assignment to the employer and the desirability of the job assignment to the personnel must be within "acceptable ranges" in minimizing the cost of the assignments. In addition to the costs, numbers representing the utility and desirability of the employer and employee respectively are shown on the arcs in Figure 15.

One way to handle these additional goals is to find first the optimal value of each goal independent of the other goals. For instance, find the optimal value of the utility function independent of the cost and desirability functions. Let u^+ denote this value. Similarly, let c^+ and d^+ denote the optimal value of the cost and desirability functions. Then add the extra linear constraints

$$\frac{u(x) - u^+}{u^+} \leq \frac{c(x) - c^+}{c^+} \tag{1}$$

and

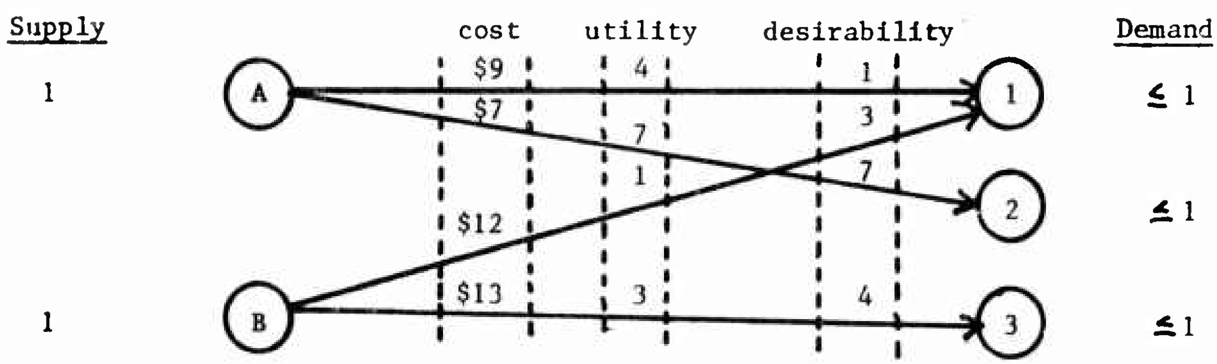
$$\frac{d(x) - d^+}{d^+} \leq \frac{c(x) - c^+}{c^+} \tag{2}$$

to the manpower problem, where u(x), d(x) and c(x) represent the utility, desirability, and cost functions. The goal is then to minimize the cost function subject to the manpower assignment constraints (network constraints) and the extra constraints. The solution will have the following property. Equation (1) will force the relative deviation of the utility function value from its optimal value u^+ to be less than or equal to the relative deviation of the cost function value from its optimal value c^+. Intuitively this is an appealing solution since it balances the goals independently of their units of measure, and forces all deviations of the goals from their optimum values to be as small as possible.

This type of problem cannot normally be transformed into a network problem. That is, the extra constraints cannot be directly incorporated into a network structure. However, it is possible to design computationally efficient solution codes for problems of this type [23, 31] . In particular, we have designed a solution code for solving pure network problems with one extra constraint which is 100 times faster than state-of-the-art commercial linear programming computer programs [23] .

Figure 15

MANPOWER ASSIGNMENT PROBLEM



VIII. CONCLUSION

The important thing to bear in mind is that it is unnecessary to be able to specify all problem characteristics in a rigorous mathematical sense at the outset of constructing a network model; often parenthetical annotations will maintain these characteristics in view while the model undergoes refinement. Once the stage is finally reached at which the crucial interrelationships are singled out, the effort to identify the best formulation and its appropriately matched solution approach can be undertaken. At this stage, the intimate coordination of modeling techniques and computer solution methods is indispensible.

This paper details several actual network applications on which we have worked during the last five years. The challenge of these applications has been an effective incentive to our efforts to develop improved solution codes with enhanced human engineering features. Accordingly, we invite those who professionally develop or apply mathematical programming techniques to feel more than welcome to share their problems and modeling needs with us, as a basis for a fertile interchange of ideas that may similarly contribute to further advances in MS/OR methodology and user-oriented software for solving real world problems.

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